Conic Sections : the intersection of a plane and a double-napped cone

How to tell which conic you are working with based on its equation…

When the equation is equal to zero, ask yourself:

1. Is there more than one squared term?
   NO – Parabola YES – Go on to question 2

2. Do the squared terms both have the same sign?
   NO – Hyperbola YES – Go on to question 3

3. Are the coefficients in front of the squared terms the same?
   NO – Ellipse YES - Circle

Examples:
Classify each of the following equations as the equation of a parabola, ellipse, circle, or hyperbola.

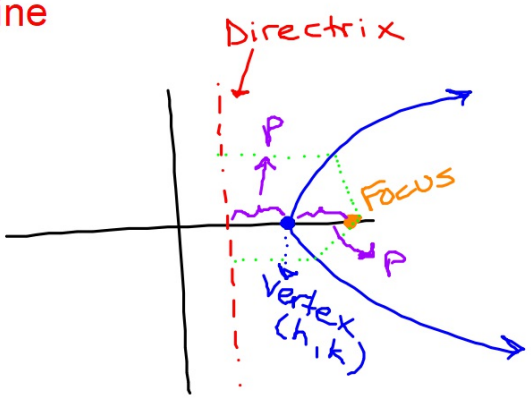
1. $3x^2 - 4x + 2 + 4y^2 - 6y + 7 = 0$ Ellipse

2. $5x^2 + 4x + 2 - 6y + 7 = 0$ Parabola

3. $2y + 5y^2 + 2 + 5x^2 - 6x - 1 = 0$ Circle

4. $2 + 3x^2 + x + 8 - 7y^2 + 2y = 0$ Hyperbola

Parabola: The set of all points (x, y) in a plane that are equidistant from a fixed line (directrix) and a fixed point (focus), not on the line



Vertex: The midpoint between the focus and the directrix (h, k)

Axis: the line passing through the vertex and focus

Standard Form:

Up / Down: $(x-h)^2 = 4p(y-k)$

Left / Right: $(y-k)^2 = 4p(x-h)$

$p \neq 0$

Changing from the general form to standard form for a parabola

Completing the square:

Example:

1. $x^2 - 8x + 11 = 0$

$-11 \quad -11$

$x^2 - 8x = -11$

$\left(-\frac{8}{2}\right)^2 = (-4)^2 = 16$

$x^2 - 8x + 16 = -11 + 16$

$(x-4)^2 = 5$

$\boxed{(x-4)^2 - 5 = 0}$

Steps:

1. Make sure "a" (leading coefficient) = 1
2. Move constant to the other side
3. Complete the Square (Add $\left(\frac{b}{2}\right)^2$ to both sides
4. Simplify both sides
5. Move constant back (set = 0)

2. $\frac{2x^2}{2} - \frac{12x}{2} + \frac{23}{2} = \frac{0}{2}$

$x^2 - 6x + \frac{23}{2} = 0$

$x^2 - 6x + 9 = -\frac{23}{2} + 9$ $\quad -\frac{23}{2} + \frac{18}{2} = \frac{-5}{2}$

$(x-3)^2 = -\frac{5}{2}$

$(x-3)^2 + \frac{5}{2} = 0$

Finding the Focus of a Parabola:

1. Covert to standard form by completing the square

2. Identify the vertex

3. Solve for "p"

4. Decide if the parabola opens up/down or left/right based on "p"

5. Add "p" to the appropriate coordinate

Examples:

Find the vertex, focus, and directrix of each parabola and sketch its graph.

up/down

1. $(x+3)^2+8(y-2)=0$

$(x+3)^2 = -8(y-2)$

$(x-h)^2 = 4p(y-k)$

$\frac{4p}{4} = \frac{-8}{4}$

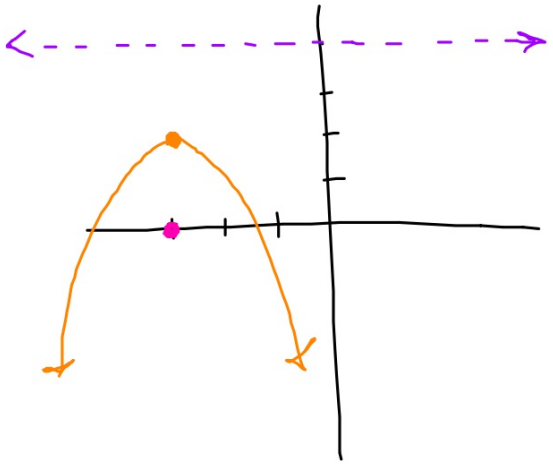
$p = -2$

down

Vertex: $(-3,2)$

Focus: $(-3,0)$

Directrix: $y=4$



2. $x^2 - 2x + 8y + 9 = 0$

$x^2 - 2x = -8y - 9$

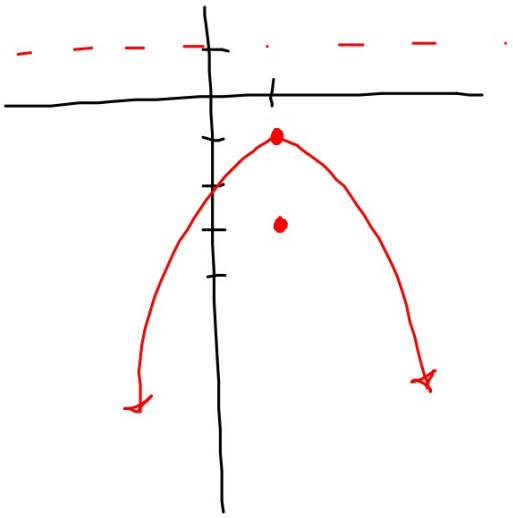
$x^2 - 2x + 1 = -8y - 9 + 1$

$(x-1)^2 = -8y - 8$

$(x-1)^2 = -8(y+1)$

$4p = -8$

$p = -2$



Vertex: $(1, -1)$

Directrix: $y = 1$

Focus: $(1, -3)$

3. $y^2 - 4y - 4x = 0$

Left/Right

$y^2 - 4y = 4x$

$y^2 - 4y + 4 = 4x + 4$

$(y-2)^2 = 4(x+1)$

$4p = 4$

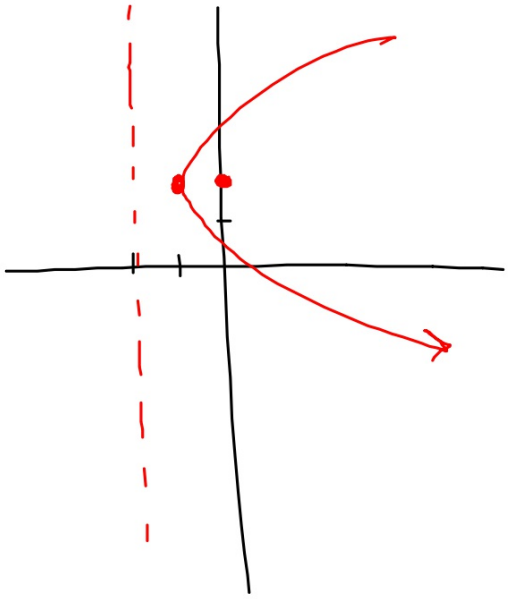
$p = 1$

Right

Vertex: $(-1, 2)$

Focus: $(0, 2)$

Directrix: $x = -2$

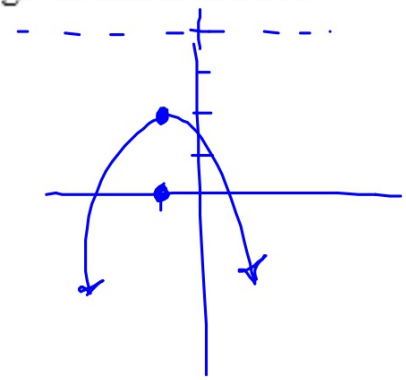


Find the standard form of the equation of a parabola with the given characteristics.

1. Vertex: (-1, 2) Focus: (-1,0)

h, k

$p = -2$



$(x-h)^2 = 4p(y-k)$

$(x+1)^2 = 4(-2)(y-2)$

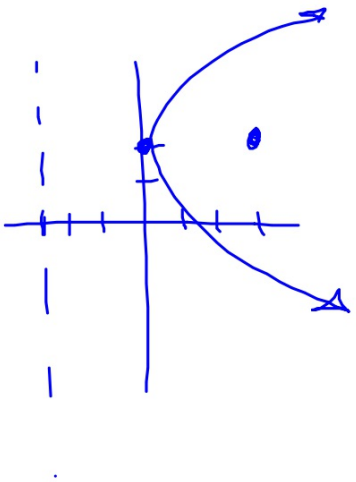
$(x+1)^2 = -8(y-2)$

2. Vertex: (0, 2) Directrix: x = -3

$(y-k)^2 = 4p(x-h)$

$(y-2)^2 = 4(3)(x-0)$

$\boxed{(y-2)^2 = 12x}$



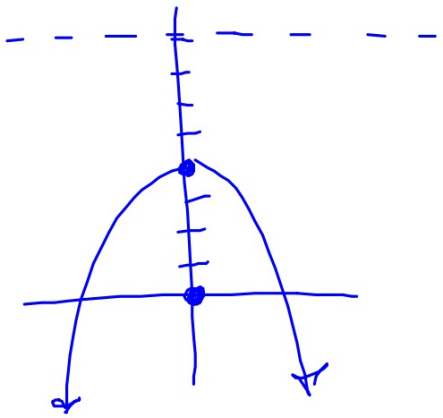
3. Focus: (0, 0) Directrix: $y = 8$

$V: (0, 4)$

$(x-h)^2 = 4p(y-k)$

$(x-0)^2 = 4(-4)(y-4)$

$\boxed{x^2 = -16(y-4)}$



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